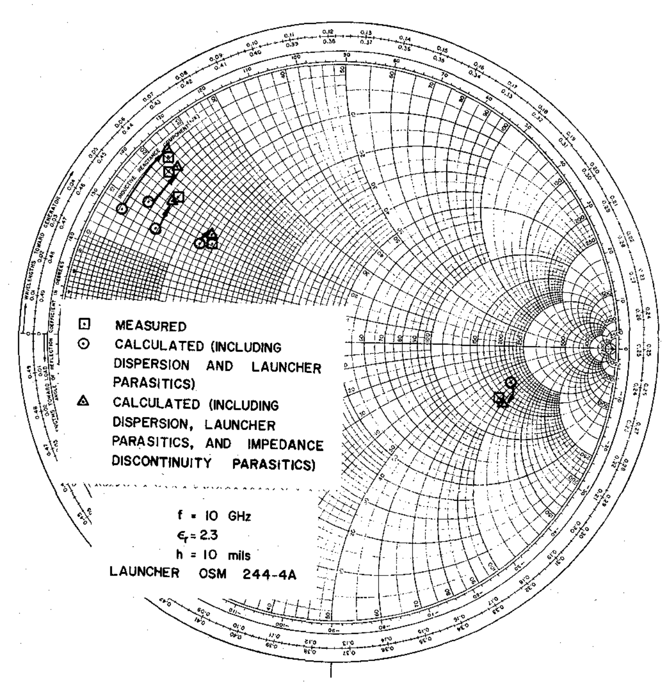
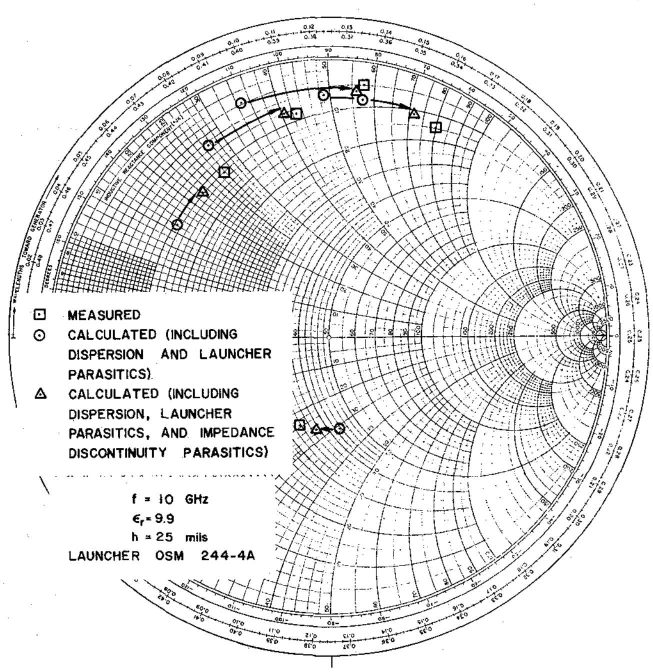


(a)



(b)

Fig. 6. Measured and calculated impedance values (including either dispersion and launcher parasitics or dispersion, launcher parasitics, and impedance discontinuity parasitics as indicated) for transformer structures on plastic and alumina substrates at 10 GHz. (a) Launcher OSM 244-4A, $\epsilon_r = 2.3$, $h = 10$ mil. (b) Launcher OSM 244-4A, $\epsilon_r = 9.9$, $h = 25$ mil.

model described by the equations is confirmed using a variety of quarter-wave transformer structures. It is shown that at X-band frequencies, the effect of the impedance discontinuity parasitic reactances is significant, much larger in fact than that of the dispersion of the effective dielectric constant.

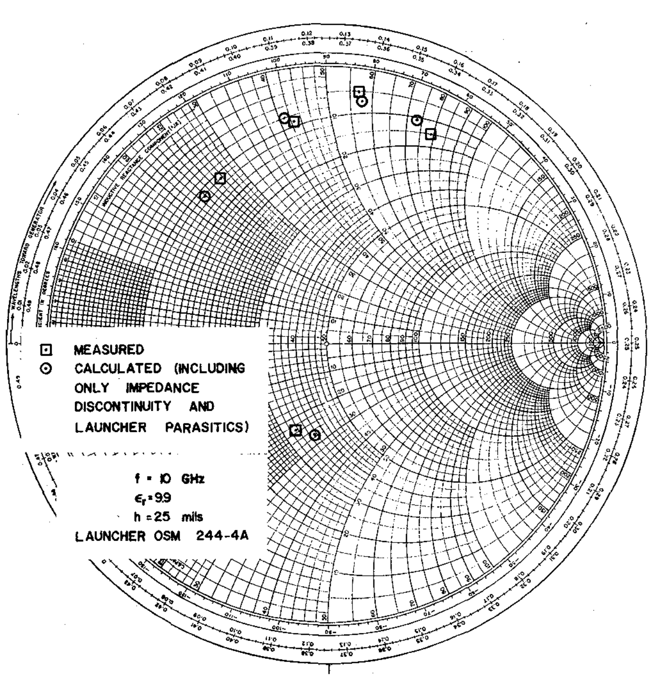


Fig. 7. Measured and calculated impedance values (including only launcher and impedance discontinuity parasitics) for a transformer structure on an alumina substrate (f = 10 GHz, ϵ_r = 9.9, h = 25 mil).

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Microwave Measurement of Dielectric Constant of Liquids and Solids Using Partially Loaded Slotted Waveguide

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***Abstract*—An accurate method is described for the measurement of the dielectric constant of liquids and solids. The dielectric material partially loads a slotted rectangular waveguide and the guide wavelength is measured for two different thicknesses of the dielectric. The guide wavelengths are related to the dielectric con-**

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stant of the material through a characteristic equation which can be solved graphically or numerically. Some experimental results are obtained and found to be in close agreement with the values of the dielectric constant available in the literature.

Introduction

The microwave methods provide a convenient means of measurement of the dielectric constant of liquids and solids. Both the cavity and the waveguide methods are used. The earlier waveguide methods used to calculate the dielectric constant of liquids and solids involve measurements of attenuation and phase shift when the waveguide is terminated in a dielectric or measurement of guide wavelength when the dielectric loads the waveguide in the H plane.

The earlier methods for accurate measurement of the dielectric constant of liquids and solids are costly and complicated. The method described here uses a commonly available slotted waveguide. The dielectric material partially fills the waveguide in the H plane. The propagation constants $(=2\pi/\lambda_g)d$ are measured along the waveguide for two different thicknesses of the dielectric slab. The dielectric constant is then related to the propagation constants through a transcendental equation which can be solved using a graphical or a numerical method. A similar method has been reported earlier by Bahl and Gupta [1] for measuring the electrical parameters of the artificial dielectrics.

Theory

Consider a rectangular waveguide with perfectly conducting walls, loaded with a dielectric slab, as shown in Fig. 1. The waveguide is filled between $y = 0$ and $y = d$ with a nonabsorbing and nonmagnetic dielectric of constant ϵ_r. This configuration has two homogeneous regions, $0 < y < d$, and $d < y < b$ and the mode of propagation is an LSM mode. The dispersion relation is given as follows [2], [3]:

$$k_{y1} \tan (k_{y1}d) = -\epsilon_r k_{y2} \tan [k_{y2}(b - d)] \tag{1}$$

where

$$k_{y1}^2 = k_0^2\epsilon_r - k_z^2 - (n\pi/a)^2; \tag{2}$$

$$k_{y2}^2 = k_0^2 - k_z^2 - (n\pi/a)^2; \tag{3}$$

k_0 the propagation constant in the free space;
k_z the propagation constant in the loaded waveguide;
n the order of occurrence of the zero of dispersion relation.

From (2) and (3)

$$k_{y1}^2 - k_{y2}^2 = k_0^2(\epsilon_r - 1). \tag{4}$$

Let for the dielectric slab of the thickness d_1, $k_{y1} = k_{y11}$, $k_{y2} = k_{y21}$, and for the thickness d_2, $k_{y1} = k_{y12}$, $k_{y2} = k_{y22}$. Then for $d = d_1$, (1) and (4) become

$$(k_{y11}d_1) \tan (k_{y11}d_1) = -\frac{\epsilon_r d_1}{b - d_1} [k_{y21}(b - d_1)] \tan [k_{y21}(b - d_1)] \tag{5}$$

and

$$k_{y11}^2 - k_{y21}^2 = k_0^2(\epsilon_r - 1). \tag{6}$$

Similarly, for $d = d_2$, (1) and (4) become

$$(k_{y12}d_2) \tan (k_{y12}d_2) = -\frac{\epsilon_r d_2}{b - d_2} [k_{y22}(b - d_2)] \tan [k_{y22}(b - d_2)] \tag{7}$$

and

$$k_{y12}^2 - k_{y22}^2 = k_0^2(\epsilon_r - 1). \tag{8}$$

Let $d_1/(b - d_1) = R_1$, $d_2/(b - d_2) = R_2$, $k_{y11}d_1 = x_1$, and $k_{y12}d_2 = x_2$. Then from (5) and (7)

$$\frac{x_1 \tan x_1}{x_2 \tan x_2} = \frac{R_1}{R_2} \frac{[k_{y21}(b - d_1)] \tan [k_{y21}(b - d_1)]}{[k_{y22}(b - d_2)] \tan [k_{y22}(b - d_2)]}. \tag{9}$$

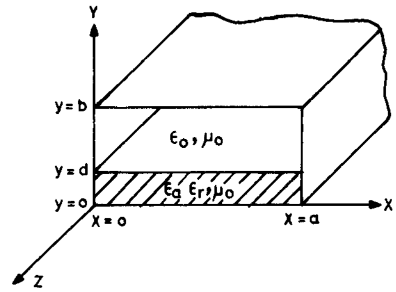

Fig. 1. Waveguide partially loaded with dielectric material.

Also from (6) and (8)

$$\left(\frac{x_2}{d_2}\right)^2 - \left(\frac{x_1}{d_1}\right)^2 = k_{y22}^2 - k_{y21}^2 \tag{10}$$

$k_{zi} = (2\pi)/(\lambda_{gi})$, $i = 1,2$ are calculated by measuring λ_g for two cases. The k_{y21} and k_{y22} are calculated from (3) for the dominant mode LSM_{11} (quasi-TE_{10}). Therefore, the right-hand sides of (9) and (10) are known quantities. Denoting the right-hand sides of (9) and (10) by c and k_d^2, respectively, (9) and (10) become

$$\frac{\cot x_2}{x_2} = c \frac{\cot x_1}{x_1} \tag{11}$$

and

$$\left(\frac{x_2}{d_2}\right)^2 - \left(\frac{x_1}{d_1}\right)^2 = k_d^2. \tag{12}$$

Equations (11) and (12) are the transcendental and the hyperbolic equations, respectively. A computer program has been written which solves (11) and (12) for x_1 and x_2. Thus k_{y11} and k_{y12} are evaluated. Since k_{y21} and k_{y22} are known by measurements, ϵ_r is evaluated with the help of (6) or (8). Final expressions for evaluation of ϵ_r are given below.

$$\epsilon_r = 1 + \frac{1}{k_0^2}\left[\frac{x_1^2}{d_1^2} - k_{y21}^2\right] \tag{13a}$$

or

$$\epsilon_r = 1 + \frac{1}{k_0^2}\left[\frac{x_2^2}{d_2^2} - k_{y22}^2\right]. \tag{13b}$$

The above formulation uses two thicknesses of the dielectric slab to calculate ϵ_r. The dielectric constant ϵ_r can also be calculated using one thickness formulation [3] with the help of (1)–(3). The experimental results for one thickness and two thickness formulations are presented and discussed in the next section.

Experimental Results

Experiments have been carried out to measure the dielectric constants of benzene, cyclohexane, and Perspex. The block schematic diagram is shown in Fig. 2. The slotted waveguide is sealed by using a thin Mylar sheet (0.005 in thick) between the two connecting flanges. The accurately measured volume of liquids is poured into

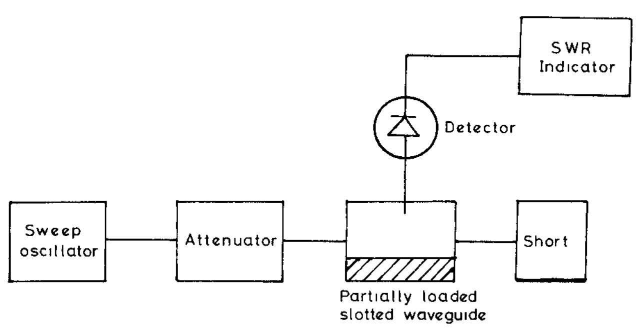

Fig. 2. Experimental setup.

TABLE I

Measured values of dielectric constant at 10 GHz

Material	Measured dielectric constant by 1-thickness method*		Measured dielectric constant by 2-thickness method	Available dielectric constant
	d=b/4	d= b/2		
Benzene (20°C)	1.47	1.79	2.27	2.285[4] (at 20°C and 9.4 GHz)
Cyclohexane (20°C)	1.43	1.66	2.04	2.024 [4] (at 20°C and 9.4 GHz)
Perspex	2.46	2.55	2.61	2.60 [5] (at 5.0 GHz)

* One thickness results are only for comparison.

the slotted waveguide such that the required thickness of the dielectric layer is obtained. The guide wavelength is measured for two thicknesses with the help of the probe in the slot and the standing wave ratio (SWR) meter. Knowing the frequency, the two thicknesses, and the corresponding wavelengths, the dielectric constant is evaluated using (13). Some of the results are given in Table I. Values of the dielectric constant are also calculated using (1)–(3) for two thicknesses separately and reported in the same table.

Discussion

The experimental values of the dielectric constants of benzene, cyclohexane, and Perspex, obtained by using the two thickness formulation, are in good agreement with the corresponding available values in the literature. The formulation in the proposed method does not require any quasi-static approximation which is needed in the cavity perturbation method. Also in the present method the problem of mode jumping does not exist. However, the propagation of higher order modes in such an H-plane loaded waveguide [2] can be avoided by careful selection of two thicknesses of the dielectric material. For example, $\epsilon_r \leq 2.66$ and $f \leq 10$ GHz, higher order modes do not propagate for $d \leq b/2$. Our results indicate that the one thickness method fails to give agreeable values of ϵ_r. It seems that there is some experimental error in the one thickness method and it gets cancelled when the two thickness method is used. The other advantage of this two thickness method is that (11) and (12) can be solved both graphically and numerically, while for the one thickness method no convenient graphical method is available. In order to make this method more practicable, tables for ϵ_r can be made for various combinations of λ_{g1} and λ_{g2}. Also the proposed method does not require a sophisticated sample holder used in other methods. It uses commonly available slotted waveguide, but the use is restricted to noncorrosive materials.

The results obtained for liquids and solids are accurate to within 0.8 percent of the values available in the literature. The guide wavelength and the thickness of the dielectric slab are normally measured to the accuracy of 0.005 cm, and an error of this magnitude corresponds to less than 1 percent in ϵ_r.

Acknowledgment

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A Finite Difference Method for the Solution of Electromagnetic Waveguide Discontinuity Problems

G. MUR

Abstract—**A finite difference method for the numerical solution of electromagnetic waveguide discontinuity problems is presented. The method of boundary relaxation is applied, using finite difference techniques in the nonuniform section of the waveguide and using a modal representation of the field in the uniform sections of the waveguide.**

To illustrate the process some two-dimensional diffraction problems in an electromagnetic waveguide with rectangular cross section are solved.

I. Introduction

Finite difference methods for the numerical solution of boundary value problems are limited to problems involving only relatively small regions, due to limitations on computer time and storage requirements. Applying the method of "boundary relaxation," Silvester and Cermak [1]–[3], Richter [4], and Sandy and Sage [5] have been able to reduce considerably the number of mesh points required and thus have reduced computing time and storage problems. The method of boundary relaxation involves the choice of an appropriate artificial boundary that limits the region to which the finite difference scheme is applied.

In this short paper an alternative method to determine the field distribution on the artificial boundary is described. Contrary to the method indicated above, it does not require the computation and storage of a large matrix and is therefore, in general, less storage and time consuming. The artificial boundary in our configuration is chosen such that the field in a suitably chosen exterior region can easily be expressed in terms of some type of modal representation of the wave function. To illustrate the process we solve some two-dimensional diffraction problems in an electromagnetic waveguide with rectangular cross section. The modal representation of the field in the exterior domain has also been used by Patwari and Davies [6] in their computation of the field scattered by conducting cylinders. However, they did not employ boundary relaxation, but used a direct method to solve the relevant system of equations.

II. Formulation of the Problem

As an example illustrating our version of the technique of boundary relaxation, we determine the scattering properties of a cylindrical obstacle and/or a cylindrical wall deformation present in a finite section of an otherwise uniform electromagnetic waveguide with rectangular cross section. To locate a point in the configuration, a right-handed Cartesian coordinate system x,y,z is introduced. The z axis is chosen parallel to the axis of the waveguide; the y axis is taken parallel to the direction of cylindricity of the obstacle and/or the wall deformation. The waveguide walls and the obstacle are assumed to be electrically perfectly conducting. The medium inside the waveguide is linear, homogeneous, isotropic, and lossless; its electromagnetic properties are characterized by a permittivity ϵ and a permeability μ. The analysis is carried out in terms of LSE and LSM fields [7], [8]. As the obstacle and/or the deformed waveguide walls are uniform in the y-direction, the total field will show the same y-dependence as the prescribed incident field. Furthermore, no coupling between LSE and LSM fields takes place.

A longitudinal cross section of the configuration to be investigated is shown in Fig. 1. The nonuniformity of the waveguide is assumed to be located in the region between the reference planes $z = z_1$ and $z = z_2$. All field quantities are assumed to vary sinusoidally in time with angular frequency ω. The complex time factor $\exp(i\omega t)$ is omitted in the formulas.

An LSE field is an electromagnetic field in the configuration for which $E_y = 0$ and $H_y \neq 0$. H_y can be written as

$$H_y = \Psi(x,z) \sin (n\pi y/b), \qquad (n = 1,2,\cdots) \tag{1}$$

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